单箱双室简支箱梁剪切变形
剪力滞效应分析

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摘 要: 基于各个翼板选取不同的最大剪切转角差为剪力滞广义位移, 考虑剪力滞和剪切变形双重效应, 应用能量变分原理推导出双室箱梁受竖向对称荷载时的截面控制微分方程组, 采用降阶法并结合边界条件导出相应的闭合解, 从力学和数学角度解释剪切变形对箱梁截面纵向应力无影响。以典型的单箱双室简支箱梁为例, 利用数值方法和本文解析解方法, 研究满跨均布力和跨中集中力荷载作用下, 跨中截面剪力滞横向分布规律和高跨比对剪力滞效应的影响规律, 研究结果表明: 顶(底)板与腹板交汇处表现为正剪力滞效应, 单室顶(底)板中心表现为负剪力滞效应; 集中力下跨中截面测点剪力滞系数随高跨比的变化分别呈线性、曲线分布, 顶(底)板与腹板交汇处测点剪力滞系数随高跨比的增大而增大, 单室顶(底)板中心测点剪力滞系数随高跨比的增大而减小。

关键词: 双室箱梁; 剪力滞效应; 剪切变形; 能量变分法; 有限元

Abstract: In view of each wing shoulder, different maximum shear angle differences are selected as the shear lag of generalized displacement. Taking the double effects of shear lag and shear deformation into consideration, the shear differential equations of twin-cell box girder under the vertical symmetrical load are deduced on the basis of energy variational principle. The corresponding closed solutions are obtained by the method of reduced order and the boundary conditions. From the point of view of mechanics and mathematics, the shear deformation has no effect on the longitudinal stress of box girder. A typical example of the twin-cell box girders, using the numerical method and the analytic solution method in this paper to study shear lag transverse distribution law and effect of high span ratio on shear lag effect of measuring point in middle span under the uniform force and concentrated force. Research shows that: the junction of the top (bottom) plates and the web plates are shown the positive shear lag effect, the middle of the top (bottom) of the single room are shown the negative shear lag effect. The variation of the shear lag coefficient with the height span ratio is the linear and the curve distribution under the concentrated force and uniform force, respectively the shear lag coefficient between the top (bottom) plates and

Analysis on shear deformation and shear–lag effect of twin–cell box girders

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the web plates increase with the increase of high span ratio. The middle of the top (bottom) of the single room decreases with the high span ratio increasing.

**Key words:** twin-cell box girder; shear-lag effect; shear deformation; Energy variational method; finite element

薄壁箱梁因其有利的受力特性而被广泛应用于现代桥梁工程中。薄壁箱梁发生竖向挠曲变形时，由腹板传递给翼缘板的剪力流使翼缘板在远离腹板处的纵向位移滞后于靠近腹板处的纵向位移，从而使箱梁翼缘板不满足平截面假设，这就是剪力滞效应\(^{1-5}\)。薄壁箱梁被广泛应用于桥梁上部结构设计当中，对其设计和计算时必须考虑剪力滞效应\(^{1,3-4}\)，但是要不要考虑剪切变形，以及剪切变形对剪力滞效应是否有影响需要进一步论证和阐述。刘世忠等\(^{6-10}\)以单室简支箱梁为例，论述了剪切变形对剪力滞效应没有影响，即剪切变形不影响箱梁截面的纵向应力。目前，大跨度、多室宽体箱梁已被广泛应用于实际桥梁建造中，考虑剪切变形对多室箱梁剪力滞效应的影响有待进一步从数学和力学角度出发，研究其受力机理和变形模式。本文从这2个方面出发，应用能量变分原理推导出箱梁截面控制微分方程组，采用降阶法求得其通解，并结合相应的边界条件给出对应的闭合解。针对一个典型的单箱双室简支箱梁为例，验证本文分析方法的精确性，从数学和力学角度解释剪切变形对箱梁纵向应力没有影响。

1 单箱双室箱梁截面控制微分方程的推导

如图1所示，XOY为整体坐标系，xoy为局部坐标系，箱梁在竖向任意荷载\(q(z)\)作用下的挠曲变形，选取最大剪切转角位移差作为剪力滞广义位移，即箱梁截面任意一点处的纵向位移表达式为

顶板:
\[
-u_1(x,y,z) = \frac{1}{h} \left[ (w' - \beta) + \cos \frac{\pi}{b_1} (x - \frac{b_1}{2}) \cdot U_1(z) \right]
\]

(2)

悬臂板:
\[
-u_2(x,y,z) = \frac{1}{h} \left[ (w' - \beta) + \cos \frac{\pi x}{2b_1} \cdot U_2(z) \right]
\]

(3)

底板:
\[
-u_3(x,y,z) = -\frac{1}{h} \left[ (w' - \beta) + \cos \frac{\pi}{b_3} (x - \frac{b_3}{2}) \cdot U_3(z) \right]
\]

(4)

腹板:
\[
u_w(z) = (w' - \beta) z
\]

(5)

其中: \(\beta(z) = \frac{\alpha Q(z)}{GA}\)。

式中: \(\alpha\) 为剪切系数，\(\alpha = \frac{A}{A_w}\); \(A\) 为箱梁截面面积；
\(A_w\) 为箱梁截面腹板面积。

令 \(\phi(z) = w(z) - \beta(z)\)

图1 箱梁截面简图

Fig.1 Box girder with cross section

顶板、悬臂板和底板的应变能表达式为
$$U_i = \frac{1}{2} \int_0^l \left[ (E\varepsilon_{i,i}^2 + G\gamma_{i,i}^2) \right] dx dy dz \quad (6)$$

其中: $\varepsilon_{i,i} = \frac{\partial u_i(x, y, z)}{\partial z}, \gamma_{i,i} = \frac{\partial u_i(x, y, z)}{\partial x}$；

$$E$$ 为弹性模量; $G$ 为剪切模量。

箱梁各个翼板的应变能:

顶板应变能:

$$\tilde{U}_1 = EI \int_0^l \left[ \frac{1}{2} \varphi^2 + \frac{1}{4} U_1^2 + \frac{2}{\pi} \varphi' U_1' \right] dz + \frac{G\pi^2 L_1}{4 b_1^2} \int_0^l U_1^2 dz \quad (8)$$

悬臂板应变能:

$$\tilde{U}_2 = EI \int_0^l \left[ \frac{1}{2} \varphi^2 + \frac{1}{4} U_2^2 + \frac{2}{\pi} \varphi' U_2' \right] dz + \frac{G\pi^2 L_2}{16 b_2^2} \int_0^l U_2^2 dz \quad (9)$$

底板应变能:

$$\tilde{U}_3 = EI \int_0^l \left[ \frac{1}{2} \varphi^2 + \frac{1}{4} U_3^2 + \frac{2}{\pi} \varphi' U_3' \right] dz + \frac{G\pi^2 L_3}{4 b_3^2} \int_0^l U_3^2 dz \quad (10)$$

腹板应变能:

$$\tilde{U}_w = \frac{EI}{2} \int_0^l \varphi^2 dz + \frac{G\pi^2 L_w}{2} \int_0^l \beta^2 dz \quad (11)$$

外力势能:

$$V = -\int_0^l q w dz \quad (12)$$

箱梁总势能:

$$\pi = \tilde{U}_1 + \tilde{U}_2 + \tilde{U}_3 + \tilde{U}_w + V \quad (13)$$

将式(8)～(12)代入式(13)可得

$$\pi = \frac{EI}{2} \int_0^l \varphi^2 dz + \frac{EI}{4} \sum_{i=1}^3 \alpha_i U_i^2 dz + \frac{2EI}{\pi} \sum_{i=1}^3 \alpha_i \varphi' U_i' dz + \frac{G\pi^2 \alpha_1}{4 b_1^2} U_1^2 + \frac{G\pi^2 \alpha_2}{4 b_2^2} U_2^2 + \frac{G\pi^2 \alpha_3}{4 b_3^2} U_3^2 + \frac{GA_w}{2} \int_0^l \beta^2 dz - \int_0^l q w dz \quad (14)$$

式中: $\alpha_i = \frac{I_i}{I}, I$ 为箱梁截面抗弯惯性矩; $q$ 为荷载集度。

将式(14)求一阶变分，并令 $\delta \pi = 0$
2.1 微分方程组的边界条件及闭合解

对于式 (26) 的求解, 采用降阶法 [12] 得到一阶线性非齐次微分方程组, 按照惯用的方法先求得齐次微分方程组的通解, 再求出非齐次微分方程组的特解, 然后将二者叠加得到非齐次微分方程组的通解, 利用各自的边界条件得到相应的闭合解。

2.1.1 箱梁受均布力时

国家标准规定的箱梁受均布力时的计算简图如图 2 (b) 所示。箱梁受均布力时的计算简图如图 2 (b) 所示。

2.2 边界条件

1) 集中力时:

\[ U_i \bigg|_{z=0} = 0, \quad U_i^{'} \bigg|_{z=0} = 0, \quad U_i^{''} \bigg|_{z=\pm a} = U_i^{''} \bigg|_{z=a}, \]

\[ U_i \bigg|_{z=a} + U_i^{'} \bigg|_{z=a} = 0 \]

2) 均布力时:

\[ U_i \bigg|_{z=0} = 0, \quad U_i^{'} \bigg|_{z=\pm a} = 0 \]

2.3 箱梁纵向应力解

求得各个翼板的最大剪切转角差, 由弹性力学原理即可获得各个翼板的纵向应力表达式。顶板:

\[ \sigma_1 = E_h u \left[ -\frac{2}{\pi} \sum_{i=1}^{3} \alpha_i U_i M E I \cdot \cos \frac{\pi x}{b_1} \right] \]

悬臂板:

\[ \sigma_2 = E_h u \left[ -\frac{2}{\pi} \sum_{i=1}^{3} \alpha_i U_i M E I + U_i^{'} \cdot \cos \frac{\pi x}{2b_1} \right] \]

底板:

\[ \sigma_3 = E_h u \left[ -\frac{2}{\pi} \sum_{i=1}^{3} \alpha_i U_i M E I + U_i^{'} \cdot \cos \frac{\pi x}{2b_1} \right] \]

由式 (26) 可以看出获得的微分方程组是关于最大剪切转角差的方程组, 与剪切变形量无关系; 式 (33) ～ (35) 获得的纵向应力解可以看出是关于
不同最大剪切转角差的函数，与剪切变形无关，即可以得知剪切变形不影响箱梁截面的纵向应力。

### 2.4 剪力滞系数

剪力滞系数 $\lambda$ 的定义为：初等梁理论计算出的应力为 $\sigma$，而实际截面上发生的应力为 $\sigma'$，则

$$\lambda = \frac{\sigma'}{\sigma}$$  (36)

### 3 算例

#### 3.1 算例基本概况

以文献[13]跨度 50 m 的混凝土单箱双室简支箱梁为例，截面尺寸、测点位置见图 3，材料 $E = 3.1 \times 10^4$ MPa，泊松比 $\mu = 1/6$。

1) 跨中集中力荷载 $P = 20$ kN；
2) 满跨均布力荷载 $q = 2$ kN/m。

#### 3.2 箱梁跨中截面剪力滞横向分布规律

应用本文获得的各个翼板的纵向应力解研究跨中截面的剪力滞系数横向分布规律，同时利用 ANSYS-shell63 建立有限元模型并获得数值解，以 ANSYS 数值解为参照，分析本文建立箱梁纵向应力表达式的精确性和合理性。

分析箱梁跨中截面剪力滞横向分布规律，如表 1，图 4~5。

<table>
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<th>测点位置</th>
<th>本文解</th>
<th>ANSYS 解</th>
<th>误差比/%</th>
<th>本文解</th>
<th>ANSYS 解</th>
<th>误差比/%</th>
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<td>悬臂板</td>
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<td>1.007</td>
<td>1.012</td>
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</tr>
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<td>1.053</td>
<td>-8.55</td>
<td>0.996</td>
<td>1.004</td>
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</tr>
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<td>1.101</td>
<td>1.103</td>
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</tr>
<tr>
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<td>0.991</td>
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<td>2.31</td>
</tr>
</tbody>
</table>

注：误差比 = (本文解 - ANSYS 解) / ANSYS 解 × 100

(a) 顶板测点剪力滞系数横向分布规律；(b) 底板测点剪力滞系数横向分布规律

**图 4** 简支梁—集中力跨中截面测点剪力滞系数横向分布规律

**Fig.4** Simply supported beam—lateral distributive law of shear lag coefficient in the mid-span cross section under the concentrated force
由表1可以看出：集中力作用时，跨中截面测点剪力滞系数误差比为-2%~0.57%；均布力作用时，跨中截面测点剪力滞系数误差比为-1.2%~0.5%；证实了本文理论的精确性。顶板、底板与腹板交汇处表现为正剪力滞，其他测点表现为负剪力滞；集中力下的剪力滞效应更明显于均布力下的剪力滞效应；这些规律与单室箱梁类似。此外由 ANSYS 数值解可以看出，边腹板处的剪力滞系数略大于中腹板处的剪力滞系数。

由图5~5可以看出：本文解与 ANSYS 解吻合程度良好，能够精确的反映双室箱梁剪力滞效应。

3.3 高跨比对剪力滞效应的影响规律

高跨比（H/L）是影响剪力滞效应较为敏感的因素之一[14]，通过调整图3箱梁截面的高跨比，利用本文解析解研究跨中截面顶板3号和5号测点和底板8号和9号测点剪力滞系数随高跨比的变化规律，并绘制变化曲线，如图6~7。

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图6~7可以看出;本文解析解与ANSYS解析解吻合程度良好。集中力下跨中截面测点剪力滞系数随高跨比的变化呈线性分布,均布力下跨中截面测点剪力滞系数随高跨比的变化呈曲线分布。不论是集中力还是均布力作用时,3号和8号测点剪力滞系数随高跨比的增大而增大;5号和9号测点剪力滞系数随高跨比的增大而减小。

4 结论

1) 单箱双室简支箱梁剪力滞横向分布规律:顶板,由腹板处向单室顶板中心递减;悬臂板,由腹板处向悬臂自由端递减;中梁,由腹板向单室中梁中心递减。顶板、底板与腹板交汇处表现为正剪力滞效应;单室顶板、底板中心表现为负剪力滞效应;此外,边腹板处剪力滞效应用于中腹板处剪力滞效应。

2) 单箱双室简支箱梁高跨比对剪力滞效应的影响规律:集中力下跨中截面测点剪力滞系数随高跨比的变化呈线性分布,均布力下跨中截面测点剪力滞系数随高跨比的变化呈曲线分布;不论是集中力还是均布力作用时,顶板、底板与腹板交汇处表现为正剪力滞系数随高跨比的增大而增大;单室顶板、底板中心测点剪力滞系数随高跨比的增大而减小。

3) 考虑剪切变形和剪力滞双重效应,利用能量变分法推导了双室箱梁截面控制微分方程组,并结合相应的边界条件导出了对应的闭合解,从力学和数学的角度进一步阐释了剪切变形对剪力滞效应无影响。通过算例论证了本文选取方法的合理性和理论的正确性。

参考文献:


